

that this conclusion should be accepted with some caution in view of the large uncertainty which is still associated with the Humphrey-Ross solution. These points are summarized in Table I.

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**Note on the Electromagnetic Current in  $SU_3$  Symmetry**

M. NAUENBERG\*

*Columbia University, New York, New York*

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The possible existence of strongly interacting particles which behave like triplets under the  $SU_3$  symmetry group adds a new contribution to the electromagnetic current, part of which transforms like a scalar under  $SU_3$ . We examine some consequences for the electromagnetic properties of baryons and mesons, considered as composite states of these triplets.

IN the  $SU_3$  scheme with baryons and mesons assigned to the octet representation<sup>1</sup> (eightfold way), the electromagnetic current generated by these particles transforms like the Gell-Mann-Nishijima combination of generators  $T_3 + \frac{1}{2}Y$ , where  $T_3$  and  $Y$  correspond to the third component of isospin and the hypercharge, respectively. Neglecting the  $SU_3$  symmetry breaking interaction, this transformation property leads to a number of relations among matrix elements of the electromagnetic current,<sup>2</sup> some of which may be compared directly with experiment.<sup>3</sup> However, the conjectured existence of particles which belong to the triplet representation of  $SU_3$ <sup>4-6</sup> will add in general a scalar contribution to the electromagnetic current, which can alter some of these relations.<sup>7</sup> Let us denote by  $\psi = (\psi_0, \psi_1, \psi_2)$  the three component field associated with, say, a fermion triplet having the charge structure  $(q, q+1, q)$  in units of  $e$ , where  $\psi_0$  and  $(\psi_1, \psi_2)$  transform respectively like  $I=0$  and  $I=\frac{1}{2}$  states under isospin rotation. Its electromagnetic current can be written in the form

$$\bar{\psi}[(q + \frac{1}{3})1 + T_3 + \frac{1}{2}Y]\gamma_\mu\psi, \tag{1}$$

where the first term transforms like a scalar under  $SU_3$ ; note that it vanishes in the case  $q = -\frac{1}{3}$  only.<sup>4</sup>

If the triplets are regarded as fundamental,<sup>4-6</sup> we expect that their charge structure determines the electromagnetic properties of the observed baryons and mesons. The derivation of these properties is a dynamical problem, and we face the usual complication of not being able to compute reliably the effects of strong interactions. Furthermore, even those relations among electromagnetic current matrix elements obtained on the basis of  $SU_3$  symmetry alone<sup>2</sup> may be violated, because of the existence of a symmetry breaking interaction. With this forewarning, we present here the results of some very simple calculations based on a model of baryons and mesons as bound states of triplets. Actually, the relations that are obtained are valid quite independently of the model, in the limit of exact  $SU_3$  symmetry. To this extent, the model is just a useful mathematical tool to derive consequences of the symmetry of the interaction. On the other hand, if the dynamical approximations are taken seriously, more detailed results emerge. One interesting possibility would be the determination of the charges of the triplets.

In the model,<sup>6</sup> the baryons are an octet bound state  $(\alpha\beta)$  of a fermion triplet  $(\alpha_0\alpha_1\alpha_2)$ , and the antiparticles

\* J. S. Guggenheim Fellow.

<sup>1</sup> M. Gell-Mann, California Institute of Technology Report No. 20, 1961 (unpublished); Phys. Rev. **125**, 1067 (1962). Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

<sup>2</sup> S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961). N. Cabibbo and R. Gatto, Nuovo Cimento **21**, 872 (1961).

<sup>3</sup> For example, the relation  $\mu_\Lambda = \frac{1}{2}\mu_N$ , where  $\mu_\Lambda$  and  $\mu_N$  are the  $\Lambda$  hyperon and the neutron magnetic moments (see Ref. 2). Experiments have been carried out by R. L. Cool, E. W. Jenkins, T. F. Kycia, D. A. Hill, L. Marshall, and R. A. Schluter, Phys. Rev. **127**, 2223 (1962),  $\mu_\Lambda = -1.5 \pm 0.5$  nuclear magnetons and W. Kernan, T. B. Novey, S. D. Warsaw, and A. Wattenberg, Phys. Rev. **129**, 870 (1963),  $\mu_\Lambda = 0.0 \pm 0.6$  nuclear magnetons.

<sup>4</sup> M. Gell-Mann, Phys. Letters **3**, 214 (1964). G. Zweig, CERN, Geneva (unpublished).

<sup>5</sup> J. Schwinger, Phys. Rev. Letters **12**, 237 (1963).

<sup>6</sup> F. Gursey, T. D. Lee, and M. Nauenberg (to be published).

<sup>7</sup> S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 958 (1961) included a scalar contribution in the electromagnetic current in deriving the relations among the baryon magnetic moments, without physical interpretation.

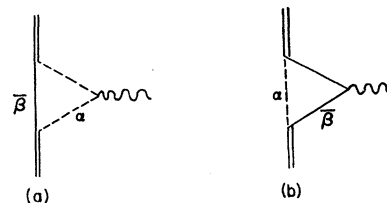


FIG. 1. A baryon  $(\alpha\beta)$  bound state (double line) interacting with a photon (wavy line) through the intermediate  $\alpha$  triplet (dashed line) in diagram (a) and  $\beta$  triplet (heavy line) in diagram (b).

of a boson triplet ( $\beta_0\beta_1\beta_2$ ), while the pseudoscalar and the vector mesons correspond to an octet ( $\alpha\bar{\alpha}$ ) and a nonet ( $\beta\bar{\beta}$ ) bound state, respectively.

To illustrate how one obtains electromagnetic properties of the bound states in this model, we consider the anomalous magnetic moments of the baryons. In a rough approximation the contribution of the  $\alpha\bar{\beta}$  intermediate state is represented by the two Feynman diagrams in Fig. 1. If we denote by  $\mu_a$  and  $\mu_b$  the moments due to the  $\alpha$  triplet (diagram a) and the  $\beta$  triplet (diagram b), respectively, we obtain the relations<sup>8</sup>

$$\begin{aligned}\mu_p &= \mu_{\Sigma^+} = (q+1)\mu_a - q\mu_b, \\ \mu_{\Sigma^-} &= \mu_{\Sigma^-} = q\mu_a - (q+1)\mu_b, \\ \mu_N &= \mu_{\Sigma^0} = q(\mu_a - \mu_b), \\ \mu_{\Sigma^0} &= [1 + (1/2q)]\mu_N, \\ \mu_{\Lambda^0} &= [1 + (1/6q)]\mu_N, \\ \mu_{\Sigma\Lambda^0} &= (1/2\sqrt{3}q)\mu_N.\end{aligned}\quad (2)$$

In the case that  $\alpha$  is a spin- $\frac{1}{2}$  fermion and  $\beta$  is a spin-0 boson, for example, it is straightforward to calculate  $\mu_a$  and  $\mu_b$  in terms of the masses of these particles and the effective coupling constant to the baryons.<sup>9</sup> It is important to note, however, that the relations among the baryon magnetic moments, Eq. (2), are a direct consequence of  $SU_3$  symmetry and the octet assignment to baryons, and do not depend on any specific model,<sup>2,7</sup> apart from the question of the existence of scalar electromagnetic currents. In the model we expect  $q$  to be an integer<sup>5,6,10</sup>; furthermore, if the triplets are singly charged,  $q = -1$  ( $q = 0$  implies zero neutron magnetic moment in this approximation). Hence, for example,  $\mu_{\Lambda} = \frac{5}{6}\mu_N$ .

We cannot justify the assumption that the above contribution is a good approximation to the observed moments, because there may also be appreciable contributions from the photons interacting with the virtual

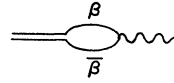


FIG. 2. A neutral vector (double line) couples to a photon (wavy line) via the intermediate ( $\beta\bar{\beta}$ ) state.

meson cloud (exchange of bound states). Although the relations given in Eq. (2) continue to remain valid, provided we neglect symmetry breaking interactions, the simple physical interpretation of the parameters, i.e.,  $q$  as charge, is then not maintained.

As another example, we consider the electromagnetic interaction of the neutral vector mesons  $\rho^0$ ,  $\omega$ , and  $\phi$ . The model<sup>6</sup> implies that these mesons should all have equal charge-conjugation quantum number  $C_p$ .<sup>11</sup> Experimental evidence indicates  $C_p = -1$ . Relations among transition amplitudes to a single-photon state can be readily obtained if we assume the dominant contribution to be given by the  $\beta\bar{\beta}$  intermediate state (see Fig. 2). We find

$$\langle \rho | \gamma \rangle : \langle \omega | \gamma \rangle : \langle \phi | \gamma \rangle = 1 : (2q+1) : \sqrt{2}q, \quad (3)$$

where  $q$  may again be interpreted as the charge of the  $I=0$  member of the  $\beta$  triplet, provided the same conditions mentioned earlier in connection with the baryon moments hold.<sup>12</sup>

Due to the lack of reliable methods for calculating the effect of strong interactions in the electromagnetic properties of baryons and mesons, we have restricted our discussion to a very simple approximation which seems rather natural in the bound-state model of these states. The approximation consists of including only certain intermediate states which have two triplets. Naturally, the results satisfy those relations based on  $SU_3$  symmetry alone, but in addition depend also on the physical properties of the conjectured triplets, which may thus be accessible to experiment.

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<sup>8</sup> I would like to thank Professor T. D. Lee for suggesting that these relations be written down for arbitrary  $q$ .

<sup>9</sup> See, for example, M. Nauenberg, Phys. Rev. **109**, 2177 (1958). The expressions for the anomalous hyperon magnetic moments can be used here, after an appropriate redefinition of constants.

<sup>10</sup> L. C. Biedenharn and E. C. Fowler, North Carolina (unpublished); H. Bacry, J. Nuyts, and L. Van Hove, CERN, Geneva (unpublished); I. S. Gerstein and K. T. Mahanthappa, University of Pennsylvania (unpublished); C. R. Hagen and A. J. Macfarlane, University of Rochester (unpublished).

<sup>11</sup> Neutral meson states with  $T_3 = Y = 0$  belonging to a multiplet which forms the basis of a self-conjugate irreducible representation of  $SU_3$ , must have the same charge-conjugation quantum number  $C$ . [T. D. Lee (private communication); see also Y. Dothan, Nuovo Cimento **30**, 399 (1964).] For example, in the case of pseudoscalar meson octet we have  $C_\pi = C_\eta$ ; experimentally  $C = 1$ . This can be shown by constructing these states out of triplets and antitriplets. Note, however, that the neutral vector mesons  $\rho_0$ ,  $\omega$ , and  $\phi$  do not belong to an irreducible representation of  $SU_3$ .

<sup>12</sup> Branching ratios for the electromagnetic decays have been calculated by Paul Singer (to be published).